

# Quantum speed-up of multiqubit open system via dynamical decoupling pulses

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We present a method to accelerate the dynamical evolution of multiqubit open system by employing dynamical decoupling pulses (DDPs) when the qubits are initially in W-type states. It is found that this speed-up evolution can be achieved in both of the weak-coupling regime and the strong-coupling regime. The physical mechanism behind the acceleration evolution is explained as the result of the joint action of the non-Markovianity of reservoirs and the excited-state population of qubits. It is shown that both of the non-Markovianity and the excited-state population can be controlled by DDPs to realize the quantum speed-up.

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## I. INTRODUCTION

Quantum speed limit (QSL) is of particular interest and has attracted much attention in tremendous areas of quantum physics and quantum information, such as quantum communication [1, 2], quantum computation [3–5], quantum metrology [6–11] and quantum optical control [12–20]. In these fields, quantum speed limit time (QSLT) is a key concept, defined as the minimum evolution time in which a system evolves from an initial state to a target state. QSLT denoted by  $\tau_{QSL}$  determines the theoretical upper bound on the speed of dynamical evolution. And by first fixing the actual driving time  $\tau$ , the shorter QSLT  $\tau_{QSL}$ , the greater the capacity for potential speedup will be. In this situation,  $\tau_{QSL} = \tau$  means that the evolution is already along the fastest path and possesses no potential capacity for further acceleration [21].

The speed-up evolution of an open quantum system is of great significance to the robustness of quantum simulators and computers against decoherence [22, 23]. Since all quantum systems are unavoidably coupled to their environments, many works have been done on the derivation of the QSLT for open system dynamics [8, 24–30] since that for isolated system dynamics [31–37]. Generally, these bounds of QSLT could be divided into two categories: Mandelstam-Tamm type based on the Cauchy-Schwarz inequality and Margolus-Levitin type based on the von Neumann trace inequality. Moreover, Deffner *et al.* demonstrated that non-Markovian effects can lead to a smaller QSLT and therefore speed up quantum evolution [27]. On the other hand, Xu *et al.* further proved that the mechanism for speedup is not only related to non-Markovianity but also to the population of excited states of the quantum system under a given driving time [28]. It is the competition between the non-Markovianity of reservoirs and the excited-state population of qubits that ultimately determines the acceleration of quantum evolution in memory environments. A question may now arise how to de-

sign a feasible and effective mechanism to achieve a acceleration process via modulating the non-Markovianity and the excited-state population . This question is of particular interest in the weak system-reservoir coupling regime, where the evolution possesses no potential capacity to accelerate without any operating on the system. However, there are few researches on this question [38, 39].

In this paper, we propose a new method to accelerate the dynamical evolution of a multiqubit open system by employing dynamical decoupling pulses (DDPs). Generally, dynamical decoupling technology is used to overcome decoherence by averaging the unwanted interaction with environment to zero [40–55]. Interestingly, we find that, DDPs can also accelerate the dynamical evolution, or in another word, reduce the time for performing an elementary logical operation. As a consequence, DDPs can greatly increase the number of operations within quantum coherence time of the system. This can help to improve the speed of quantum computers and communication channels. Meanwhile, compared with a single qubit system, the multiqubit system has more abundant structures and would have more extensive applications in quantum information processing. This is the main motivation of our present work.

In this paper, we consider a multiqubit open quantum system in which each qubit only interacts with its own reservoir, there is no direct interaction among qubits, and all reservoirs are independent with each other. For simplicity, we take the initial states of multiqubit system under our consideration to be W-type states. We find that the QSLT has nothing to do with the number of qubits and the initial-state parameters, but it can be modulated by the number of DDPs to realize the acceleration of quantum evolution. And this speed-up evolution assisted by DDPs can be achieved for both the weak system-reservoir coupling case and the strong system-reservoir coupling case. In order to reveal the essential reason of the acceleration, the roles of DDPs in the non-Markovianity and the excited-state population are studied in detail.

The paper is organized as follows. In Sec. II, we present the physical model under our consideration and derive the dynamics of multiqubit open system under DDPs. In Sec. III, we investigate the effect of DDPs on the evolution speed of

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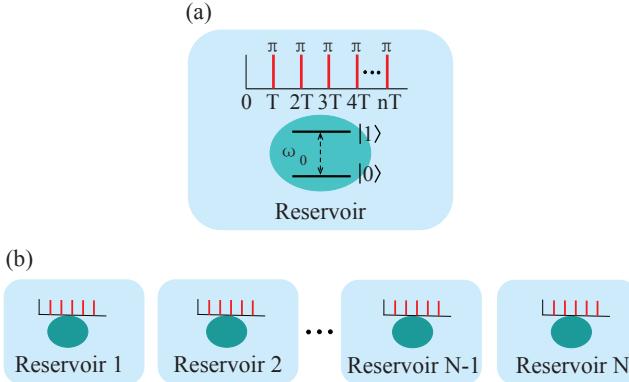


FIG. 1: (Color online) Schematic diagram of the physical system considered in this work. (a) A qubit interacts with a reservoir in the presence of periodic dynamical decoupling pulses (DDPs). The energy separation of the qubit is  $\omega_0$ , and the periodic dynamical decoupling is implemented by a train of instantaneous  $\pi$  pulses applied at the time  $T, 2T, 3T, \dots$ , and  $nT$ . (b)  $N$  pairs of independent qubit-reservoirs in which DDPs are applied to each qubit.

the multiqubit system and indicate the speed-up mechanism of the dynamic evolution. Finally, we conclude our paper with discussions and remarks in the last section.

## II. PHYSICAL MODEL AND SOLUTION

We consider  $N$  independent qubits interacting with their own reservoirs locally, and each qubit is controlled by a train of DDPs, as showed in Fig. 1. The dynamics of the  $N$ -qubit open system can be obtained by the use of time evolution of each qubit-reservoir pair with the following Hamiltonian [53]

$$H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k g_k (\sigma_+ a_k + a_k^\dagger \sigma_-) + \frac{\pi}{2} \sum_{n=1}^{\infty} \delta(t - nT) \sigma_z, \quad (1)$$

where  $\omega_0$  is the energy difference between the excited state  $|1\rangle$  and the ground state  $|0\rangle$  of the qubit,  $\sigma^\pm$  are the Pauli raising and lowering operators, and  $a_k^\dagger (a_k)$  is the creation (annihilation) operator of the  $k$ th mode of the reservoir with the mode frequency  $\omega_k$ . The coupling strength between the qubit and its reservoir is denoted by  $g_k$ , which is further characterized by the effective Lorentzian spectral density [56]

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2}, \quad (2)$$

where  $\gamma_0$  is the Markovian decay rate,  $\lambda$  is the spectral width. Typically,  $\gamma_0 < \lambda/2$  corresponds to the weak system-reservoir coupling regime, while  $\gamma_0 > \lambda/2$  corresponds to the strong system-reservoir coupling regime. Without any operation on the system, the dynamics is Markovian in the weak-coupling regime, while the dynamics is non-Markovian in the strong-coupling regime.

The last term in Eq. (1) corresponds to the dynamical decoupling control with a train of ideal instantaneous  $\pi$  pulses.  $T$  is the time interval between two consecutive pulses. Assuming that the width of each pulse is sufficiently short, it can be treated as  $\delta$  function. Under the action of each pulse, the qubit will rotate around the  $z$  axis by  $\pi$ . Then under the unitary transformation  $U(t) = \hat{T} \exp[-i \int_0^t dt_1 \frac{\pi}{2} \sum_{n=1}^{\infty} \delta(t_1 - nT) \sigma_z]$ , Hamiltonian in Eq. (1) gives rise to the following effective Hamiltonian

$$H_{\text{eff}} = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k^\dagger a_k + \sum_k (-1)^n g_k (\sigma_+ a_k + a_k^\dagger \sigma_-), \quad (3)$$

where  $n = [t/T]$  is the number of applied pulses within the driving time  $t$ , denoted by the integer part of  $t/T$ . From the effective Hamiltonian in Eq. (3), we can see that DDPs only changes the sign of the qubit-reservoir coupling strength periodically. This is the physical principle why enough DDPs can eliminate the effective coupling between the qubit and its reservoir, and hence the decoherence of open system can be efficiently suppressed.

Under the single-excitation approximation in a pair of qubit-reservoir, the reduced density matrix of the qubit at the time  $t$  can be exactly solved as

$$\rho_S(t) = \begin{pmatrix} \rho_{11}(0) \kappa_t^2 & \rho_{10}(0) \kappa_t \\ \rho_{10}^*(0) \kappa_t^* & 1 - \rho_{11}(0) \kappa_t^2 \end{pmatrix}, \quad (4)$$

or in the form of Kraus operators

$$\rho_S(t) = \sum_{i=1,2} K_i \rho_S(0) K_i^\dagger, \quad (5)$$

where the two Kraus operators are given by

$$K_1 = \kappa_t |1\rangle \langle 1| + |0\rangle \langle 0|, \quad (6a)$$

$$K_2 = \sqrt{1 - \kappa_t^2} |0\rangle \langle 1|. \quad (6b)$$

When  $t \in [nT, (n+1)T]$ , the general solution of  $\kappa_t$  reads [53]

$$\kappa_t = \begin{cases} e^{-\lambda t/2} [2\Delta_n F_{1n} + (1 + \lambda \Delta_n) F_{2n}], & \lambda = 2\gamma_0, \\ e^{-\lambda t/2} [A_n \cosh(\Delta_n d) + B_n \sinh(\Delta_n d)], & \lambda \neq 2\gamma_0, \end{cases} \quad (7)$$

which is a real number. The coefficients on the right-hand side of Eq. (7) are given by

$$F_{1n} = \frac{\lambda^2 T (\mu_+^n - \mu_-^n)}{4 \sqrt{(\lambda T)^2 + 4}}, \quad F_{2n} = \frac{\mu_+^n + \mu_-^n}{2} + \frac{4F_{1n}}{\lambda^2 T}, \quad (8a)$$

$$A_n = \alpha_+ m_+^n + \alpha_- m_-^n, \quad B_n = \beta_+ m_+^n + \beta_- m_-^n, \quad (8b)$$

where we have introduced the following parameters

$$\Delta_n = \frac{t - nT}{2}, \quad \mu_\pm = \frac{1}{2} \left[ \lambda T \pm \sqrt{(\lambda T)^2 + 4} \right], \quad (9a)$$

$$\alpha_\pm = \frac{1}{2} \left[ 1 \pm \frac{\cosh(\frac{Td}{2})}{\xi} \right], \quad \beta_\pm = \frac{\alpha_\pm [m_\pm - \cosh(\frac{Td}{2})]}{\sinh(\frac{Td}{2})}, \quad (9b)$$

with  $d = \sqrt{\lambda^2 - 2\lambda\gamma_0}$ ,  $\xi = \sqrt{1 + \left[\frac{\lambda}{d} \sinh\left(\frac{Td}{2}\right)\right]^2}$ , and  $m_{\pm} = \frac{\lambda}{d} \sinh\left(\frac{Td}{2}\right) \pm \xi$ .

From Eq. (4), we can find that the excited-state population of the qubit at the time  $t$  scaled by the excited-state population of the qubit at the initial time (i.e., the ratio of the excited-state population of the qubit between at the time  $t$  and at the initial time) can be written as the following simple expression

$$P_t = \kappa_t^2, \quad (10)$$

where  $\kappa_t$  is given by Eq. (7). It should be noted that what Eqs. (4) or (5) describe is an amplitude damping channel model. From Eqs. (4) we can see that  $P_t$  reflects not only the decay of excited-state population, but also the decay of quantum coherence of the qubit.

In what follows, we will focus on the situation of  $\lambda \neq 2\gamma_0$ . In this case,  $P_t$  becomes

$$P_t = e^{-\lambda t} [A_n \cosh(\Delta_n d) + B_n \sinh(\Delta_n d)]^2, \quad (11)$$

which indicates that  $P_t$  can be modulated by the parameters of coupling spectrum  $\gamma_0$  and  $\lambda$ , as well as the number of DDPs  $n$ .

Because every pair of qubit-reservoir is independent and identical, the dynamics of the  $N$ -qubit open system can be straightforwardly given by

$$\rho_t = \sum_{\mu_1, \dots, \mu_N=1,2} [\otimes_{j=1}^N K_{\mu_j}(t)] \rho_0 [\otimes_{j=1}^N K_{\mu_j}^\dagger(t)], \quad (12)$$

where  $K_{\mu_j}$  ( $\mu_j = 1, 2$ ) denotes the Kraus operator for the  $j$ th qubit in Eq. (6). Having obtained the density operator at any time, one can investigate the QSL of the  $N$ -qubit open system as described in the following sections.

### III. QUANTUM SPEED-UP VIA DDPS

In this section, we study the quantum speed-up of dynamic evolution of the  $N$ -qubit open system by employing DDPs. More concretely, we study the effect of DDPs on the QSLT of this system. This is because that by first fixing the actual driving time, a decrease in the QSLT results in an increased ability to speed-up, whereas an increase in the QSLT means the decrease of the potential speed-up capacity.

Now we begin with the definition of the QSLT in an open system. The QSLT is defined as the minimum time of a system evolution from an initial state  $\rho_0$  to a target state  $\rho_\tau$ , which is governed by the time-dependent master equation  $L_t \rho_t = \dot{\rho}_t$ . With the help of the von Neumann trace inequality and the Cauchy-Schwarz inequality, the QSLT reads [27]

$$\tau_{QSL} = \frac{|1 - f_\tau|}{\min\{E_1^\tau, E_2^\tau, E_\infty^\tau\}}, \quad (13)$$

where  $E_p^\tau = \frac{1}{\tau} \int_0^\tau dt \|L_t \rho_t\|_p$ , and  $\|A\|_p = (\lambda_1^p + \dots + \lambda_n^p)^{1/p}$  denotes the Schatten  $p$  norm with descending singular values of operator  $A$ ,  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ . The singular values of

an operator  $A$  are defined as the eigenvalues of  $\sqrt{A^\dagger A}$ . For a Hermitian operator, its singular values are given by the absolute value of its eigenvalues. And the fidelity between an initial pure state  $\rho_0 = |\psi_0\rangle\langle\psi_0|$  and a target state  $\rho_\tau$  is defined as  $f_\tau = \langle\psi_0|\rho_\tau|\psi_0\rangle$ . Moreover, the authors in Ref.[27] further proved that the ML-type bound based on the operator norm ( $p = \infty$ ) provides the sharpest bound on the QSLT, so we will use the following bound to derive the QSLT of the  $N$ -qubit open system,

$$\tau_{QSL} = \frac{\tau |1 - f_\tau|}{\int_0^\tau \lambda_1 dt}. \quad (14)$$

Clearly, we can obtain the QSLT by calculating the fidelity  $f_\tau$  and the operator norm of  $L_t \rho_t$ .

For simplicity, here we assume that the  $N$ -qubit system is initially prepared in the W-type states,

$$|\psi_0\rangle = \alpha_1 |100\dots0\rangle + \alpha_2 |010\dots0\rangle + \dots + \alpha_N |000\dots1\rangle, \quad (15)$$

where the normalization coefficients satisfy  $\sum_{j=1}^N |\alpha_j|^2 = 1$ . Submitting  $\rho_0 = |\psi_0\rangle\langle\psi_0|$  into Eq. (12), the reduce density matrix of the  $N$ -qubit system at the time  $t$  reads

$$\rho_t = P_t |\psi_0\rangle\langle\psi_0| + (1 - P_t) |0\dots0\rangle\langle0\dots0|, \quad (16)$$

where  $P_t$  is given by Eq. (11), and the derivation of Eq. (16) is shown in Appendix A. Hence, the fidelity between the initial W-type states  $\rho_0 = |\psi_0\rangle\langle\psi_0|$  and the final state  $\rho_\tau$  is found to be

$$f_\tau = P_\tau, \quad (17)$$

which means that the fidelity between the initial W-type states and the final state is equal to the excited-state population in the final state.

Meanwhile, Eq. (16) makes  $L_t \rho_t = \dot{P}_t (|\psi_0\rangle\langle\psi_0| - |0\dots0\rangle\langle0\dots0|)$ , and  $(L_t \rho_t)^\dagger = L_t \rho_t$ . Thus, the operator norm of  $L_t \rho_t$  can be obtained as the absolute value of its eigenvalues,

$$\lambda_1 = |\dot{P}_t|. \quad (18)$$

Then, using Eq. (14), one can get the expression of the QSLT

$$\tau_{QSL} = \frac{\tau(1 - P_\tau)}{1 - P_\tau + 2 \int_{0, \dot{P}_t > 0}^\tau \dot{P}_t dt}, \quad (19)$$

which reflects the speed of the dynamic evolution from the initial W-type states  $|\psi_0\rangle$  to the final state  $\rho_\tau$  by an actual evolution time  $\tau$ .

Interestingly, we note that the QSLT has nothing to do with the number of qubits  $N$  and the initial-state-parameter  $\alpha_j$  [21]. This is largely because the characteristic of dynamics with the initial W-type state, as shown in Eq. (16). According to Eq. (11) and Eq. (19), the QSLT could be modulated by the parameters of coupling spectrum  $\gamma_0$  and  $\lambda$ , as well as the number of DDPs  $n$  within the actual evolution time  $\tau$ . Thus, the number of DDPs can be used as a control parameter to improve

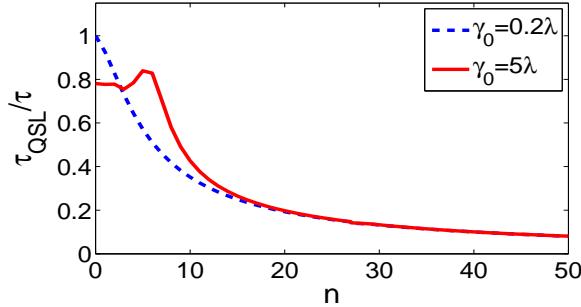


FIG. 2: (Color online) The QSLT versus the number of DDPS  $n$  for a fixed actual driving time. The dash line and the solid line correspond to  $\gamma = 0.2\lambda$  (in the weak-coupling regime) and  $\gamma = 5\lambda$  (in the strong-coupling regime), respectively. Other parameters are chosen as  $\lambda\tau = 10$  and  $\lambda = 1$ .

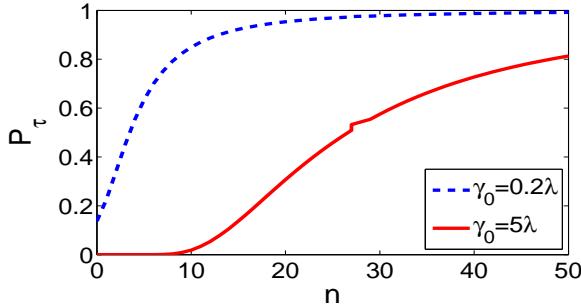


FIG. 3: (Color online) The excited-state population in the final state  $P_\tau$  as a function of the number of DDPS  $n$ . The dash line and the solid line correspond to  $\gamma = 0.2\lambda_0$  (in the weak-coupling regime) and  $\gamma = 5\lambda_0$  (in the strong-coupling regime), respectively. Other parameters are chosen as  $\lambda\tau = 10$  and  $\lambda = 1$ .

the speed of evolution. We shall numerically investigate the quantum speed-up in the following.

In order to observe the speed-up effect induced by DDPS, let's focus on the QSLT  $\tau_{QSL}$  for a fixed actual evolution time of the system  $\tau$ . When the actual evolution time  $\tau$  is fixed, for the QSLT ratio  $\tau_{QSL}/\tau < 1$  the lower ratio  $\tau_{QSL}/\tau$ , equivalently, the much shorter  $\tau_{QSL}$ , the greater the capacity for potential speedup. And in the case that the actual evolution time equals the QSLT, i.e.,  $\tau_{QSL}/\tau = 1$ , the evolution is already along the fastest path and there is no likelihood of further acceleration.

In Fig. 2 we plot the QSLT  $\tau_{QSL}$  given by Eq. (19) versus the number of DDPS  $n$  for a fixed actual driving time  $\tau$ . From this figure, we can see that in the weak-coupling regime ( $\gamma_0 = 0.2\lambda$ , the dash line), the QSLT monotonously decreases with the increase of the number of DDPS  $n$ . This clearly indicates the quantum speedup effect induced by DDPS. However, in the strong-coupling regime ( $\gamma_0 = 5\lambda$ , the solid line), the situation is different. From Fig. 2 we can see that the QSLT in the strong-coupling regime first fluctuates in a small  $n$  region and then decreases continuously with the increase of  $n$  in the same as the weak-coupling regime. That is to say, there may exist the speed-down effect in the strong-coupling regime when the number of DDPS is small. But anymore, only if the num-

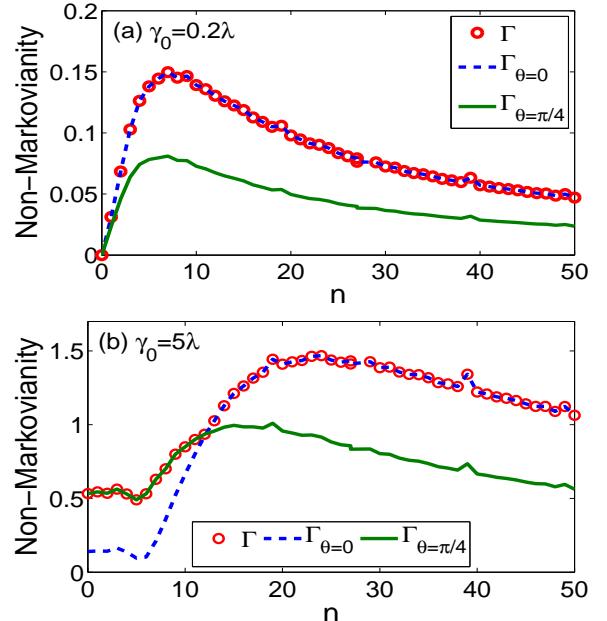


FIG. 4: (Color online) The non-Markovianity in a pair of qubit-reservoir for the fixed driving time  $\tau$  as a function of the number of DDPS  $n$ . The circle lines represent the non-Markovianity measure  $\Gamma$ . The dashed lines and solid lines correspond to the non-Markovianity components  $\Gamma_{\theta=0}$  and  $\Gamma_{\theta=\pi/4}$ , respectively. Parameters are chosen as  $\gamma_0 = 0.2\lambda$  for the weak-coupling regime,  $\gamma_0 = 5\lambda$  for the strong-coupling regime,  $\lambda\tau = 10$ , and  $\lambda = 1$ .

ber of DDPS is large enough, DDPS could induce the shorter QSLT in both the weak-coupling regime and strong-coupling regime, so as to enhance the capacity for acceleration. This accelerating effect is especially obvious in the weak-coupling regime: in the absence of DDPS (i.e.,  $n = 0$ ), the actual evolution time achieves the QSLT, and there is no likelihood of acceleration.

In order to understand the physical mechanism of the quantum speed-up assisted by DDPS, we turn to focus on the non-Markovianity of the reservoir within the driving time for a pair of qubit-reservoir [57] and the excited-state population of the qubits. Physically speaking, an increase of the distinguishability of a pair of states during any time interval implies the emergence of the non-Markovianity. This can be interpreted as a flow of information from the reservoir back to the qubit system. In Appendix B, we have derived the non-Markovianity measure  $\Gamma$  characterized in terms of the amount of information exchanged between the qubit and its reservoir [57]. By use of the optimal initial-state pair of the qubit  $|\psi_1(0)\rangle = \cos\theta|1\rangle + \sin\theta\exp(i\varphi)|0\rangle$  and  $|\psi_2(0)\rangle = \sin\theta|1\rangle - \cos\theta\exp(i\varphi)|0\rangle$  with  $\theta \in [0, \pi/2]$ ,  $\varphi \in [0, 2\pi]$ , the non-Markovianity measure in the time interval  $\tau$  can be expressed as

$$\Gamma = \max\{\Gamma_{\theta=0}, \Gamma_{\theta=\pi/4}\}, \quad (20)$$

where the two non-Markovianity components  $\Gamma_{\theta=0}$  and  $\Gamma_{\theta=\pi/4}$

are given by

$$\Gamma_{\theta=0} = \int_{0, \dot{P}_t > 0}^{\tau} \dot{P}_t dt, \quad \Gamma_{\theta=\pi/4} = \int_{0, \dot{P}_t > 0}^{\tau} \frac{\dot{P}_t}{2\sqrt{P_t}} dt. \quad (21)$$

Here the optimal initial-state pair has been numerically proved to be either  $\{|0\rangle, |1\rangle\}$  ( $\theta = 0$ ) or  $\{|+\rangle, |-\rangle\}$  ( $\theta = \pi/4$ ) [58–61]. We can see that the non-Markovian dynamics appears only when  $\dot{P}_t > 0$ , and the QSLT in Eq. (19) can be reduced to

$$\tau_{QSL} = \frac{\tau}{1 + \frac{2\Gamma_{\theta=0}}{1 - P_\tau}}, \quad (22)$$

which indicates that the QSLT is determined by two quantities: the non-Markovianity  $\Gamma_{\theta=0}$  with the initial-state pair  $\{|0\rangle, |1\rangle\}$  within the evolution time and the excited-state population in the final state  $P_\tau$  in Eq. (11). Hence, the memory effect of reservoir and the excited-state population become two essential factors for speeding up the dynamical evolution.

As an illustration, we plot the influence of the number of DDPs  $n$  on the excited-state population in the final state  $P_\tau$  of the qubit and the non-Markovianity measure of the reservoir  $\Gamma$  in Fig. 3 and Fig. 4, respectively. On one hand, from Fig. 3 we can see that in the weak-coupling case (the dashed line), the value of  $P_\tau$  of the qubit rises sharply under the action of DDPs in the small  $n$  regime, and it increases slowly in the large  $n$  regime. In particular,  $P_\tau$  can approach to unity. These excited-state population changes well match with the QSLT changes as shown in Fig. 2. On the other hand, from Fig. 4(a) we can see that the non-Markovianity is very weak with the maximum  $\Gamma_{max} \approx 0.15$ . This means the environment of the multiqubit open system under the action of DDPs is a weak non-Markovian reservoir. Hence, we can conclude that, although the non-Markovianity of reservoir is the necessary condition for speeding up [21, 28, 38], the excited-state population is the dominant mechanism of the quantum speed-up in the weak-coupling case. It is through controlling the excited-state population of the qubits that DDPs can accelerate the dynamic evolution of the quantum system under our consideration.

In the strong-coupling case (i.e.,  $\gamma_0 = 5\lambda$ ), the influence of the number of DDPs  $n$  on the excited-state population of the qubit  $P_\tau$  and the non-Markovianity measure of the reservoir  $\Gamma$  is different from the weak-coupling case. The solid line in Fig. 3 reflects changes of the  $P_\tau$  with respect to the number of DDPs while the circle line in Fig. 4(b) describes changes of the non-Markovianity measure with respect to the number of DDPs. On one hand, from Fig. 3, we can see that the qubit almost remains in the zero excited-state population for the small number of DDPs. This means that the excited-state population does not affect the speed of the dynamic evolution for the small number of DDPs. On the other hand, comparing Fig. 3 with Fig. 4(b) for the strong-coupling case, we can find that the QSLT changing rule under the small  $n$  condition well matches with that of the non-Markovianity measure. So the non-Markovianity plays the decisive role on accelerating the dynamic evolution for the small  $n$  case in the strong-coupling regime. From Fig. 3 and Fig. 4(b), we can see that both of the  $P_\tau$  and the non-Markovianity vary soundly with the number

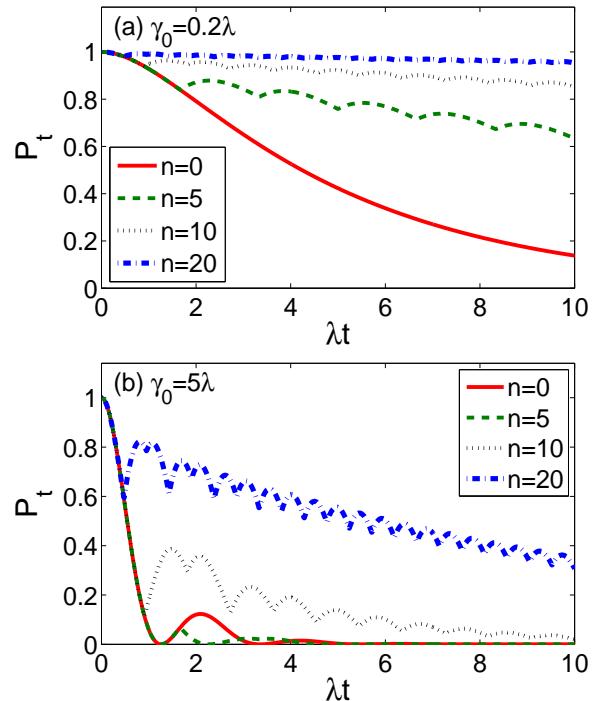


FIG. 5: (Color online) The dynamics of  $P_t$  for different numbers of DDPs:  $n=0$  (solid lines),  $n=5$  (dashed lines),  $n=10$  (dot lines),  $n=20$  (dot-dashed lines). (a) The case of the weak-coupling regime ( $\gamma_0 = 0.2\lambda$ ); (b) The case of the strong-coupling regime ( $\gamma_0 = 5\lambda$ ).  $\lambda = 1$ .

of DDPs in the large  $n$  regime of  $n > 10$ . Comparing Fig. 2 with Fig. 3 and Fig. 4(b), for the strong-coupling case we can find that both of the excited-state population and the non-Markovianity play an important role for the quantum speedup, it is the competition between the excited-state population and the non-Markovianity that takes the responsibility to accelerate quantum evolution in the large  $n$  regime of DDPs.

Finally, we discuss the control mechanism of the non-Markovianity of a reservoir by the use of DDPs. From Fig. 4 we can see that on the whole, the non-Markovianity increase firstly and then decrease later with increasing DDPs number  $n$ . This can be explained by the effect of DDPs on the dynamics of each qubit. As shown in Fig. 5, the excited-state population of the qubit  $P_t$  fast oscillates as the number of DDPs  $n$  increases. Overall, DDPs not only makes the effective coupling between each qubit and its reservoir decrease, but also makes the information exchange between them more rapidly. It is the competition between these two effects that leads to the nonmonotonic behavior of  $\Gamma$  as a function of  $n$ . Moreover, contrasting the two subfigures Fig. 4(a) and Fig. 4(b), we can see that the value of  $\Gamma$  in strong-coupling regime is greater than that in weak-coupling regime, while in the weak-coupling regime the dynamics will be Markovian ( $\Gamma = 0$ ) in the absence of DDPs ( $n = 0$ ). Besides, the optimal initial-state pairs are also different in two regimes for calculation of the measure of non-Markovianity. In the weak-coupling case, the optimal initial-state pair is proved to be  $\{|1\rangle, |0\rangle\}$  ( $\Gamma = \Gamma_{\theta=0}$ ), while in the strong-coupling case, the optimal initial-state pair

changes from  $\{|+\rangle, |-\rangle\}$  ( $\Gamma = \Gamma_{\theta=\pi/4}$ ) to  $\{|1\rangle, |0\rangle\}$  ( $\Gamma = \Gamma_{\theta=0}$ ) as  $n$  is increased. But anyway,  $\Gamma_{\theta=0}$  reflects the measure of non-Markovianity.

#### IV. CONCLUSIONS

In conclusion, we have studied the effects of DDPs on the QSLT of the  $N$ -qubit open system when the  $N$ -qubits are initially in the W-type states. We have found that DDPs can be used to accelerate quantum evolution of multiqubit open system in both of the weak-coupling and the strong-coupling regimes. In the case of the weak-coupling between qubits and reservoirs, it has been shown that the capacity of the quantum acceleration monotonously increases with the number of DDPs. In the case of the strong-coupling between qubits and reservoirs, it is indicated that the quantum speed-up capacity is the same as that in the weak-coupling case when the number of DDPs is large enough. While when the number of DDPs is small, it has been found that quantum speed-down may happen in the strong-coupling regime between qubits and reservoirs. The essential physical mechanism for the speed-up evolution is that the non-Markovianity of reservoirs and the excited-state population of the qubits jointly determine the QSLT. Under the action of DDPs, the non-Markovianity of reservoirs and the excited-state population of the qubits vary so as to lead to the quantum speed-up of the multiqubit open system. The non-Markovianity of reservoir is the necessary condition for speeding up in both of the weak- and strong-coupling regimes. The excited-state population is the dominant mechanism of the quantum speed-up in the weak-coupling case while the non-Markovianity of reservoir is the dominant mechanism of the quantum speed-up in the strong-coupling case when the number of DDPs is small. Our findings in the present paper may lead to development of effective methods for the speedup evolution of multiqubit open systems. These findings in the present paper could prove useful to accelerate quantum evolution of multiqubit open systems. In future work we will explore the possibility of realizing the quantum speed-up for a more broader class of initial states, interacted multiqubits, more complicate environments by the use of DDPs.

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#### Appendix A: Derivation of Eq. (16)

In this appendix, we present a detailed derivation of quantum state of the  $N$ -qubit system at an arbitrary time, i.e., the quantum state given by Eq. (16). When the  $N$ -qubit system is initially prepared in the W-type states

$$\begin{aligned} |\psi_0\rangle &= \alpha_1 |100\dots0\rangle + \alpha_2 |010\dots0\rangle + \dots + \alpha_N |000\dots1\rangle \\ &= \sum_{j=1}^N \alpha_j |j\rangle, \end{aligned} \quad (\text{A1})$$

where  $|j\rangle$  means that only  $j$ th qubit is in the state  $|1\rangle$  and the other  $N-1$  qubits are in the state  $|0\rangle$ . Then the density operator of the initial state can be written as

$$\begin{aligned} \rho_0 &= |\psi_0\rangle\langle\psi_0| \\ &= \sum_{j=1}^N |\alpha_j|^2 |j\rangle\langle j| + \sum_{j\neq k} \alpha_j \alpha_k^* |j\rangle\langle k| \end{aligned} \quad (\text{A2})$$

For the single qubit case, making use of Eq. (6) it is straightforward to show

$$\sum_{i=1,2} K_i |0\rangle\langle 0| K_i^\dagger = |0\rangle\langle 0|, \quad (\text{A3a})$$

$$\sum_{i=1,2} K_i |0\rangle\langle 1| K_i^\dagger = \kappa_t |0\rangle\langle 1|, \quad (\text{A3b})$$

$$\sum_{i=1,2} K_i |1\rangle\langle 0| K_i^\dagger = \kappa_t |1\rangle\langle 0|, \quad (\text{A3c})$$

$$\sum_{i=1,2} K_i |1\rangle\langle 1| K_i^\dagger = \kappa_t^2 |1\rangle\langle 1| + (1 - \kappa_t^2) |0\rangle\langle 0|. \quad (\text{A3d})$$

For the  $N$ -qubit case, by the use of Eq. (A3) we can obtain the actions of the Kraus operators on  $|j\rangle\langle k|$  ( $j, k = 1, 2, 3, \dots, N$ ) as follows

$$\sum_{\mu_1, \mu_2, \dots, \mu_N} \left[ \otimes_{i=1}^N K_{\mu_i}(t) \right] |j\rangle\langle k| \left[ \otimes_{i=1}^N K_{\mu_i}^\dagger(t) \right] = \kappa_t^2 |j\rangle\langle k|, \quad (j \neq k) \quad (\text{A4a})$$

$$\sum_{\mu_1, \mu_2, \dots, \mu_N} \left[ \otimes_{i=1}^N K_{\mu_i}(t) \right] |j\rangle\langle j| \left[ \otimes_{i=1}^N K_{\mu_i}^\dagger(t) \right] = \kappa_t^2 |j\rangle\langle j| + (1 - \kappa_t^2) |0\dots0\rangle\langle 0\dots0|. \quad (\text{A4b})$$

Making use of Eq. (A2), Eq. (A4) and Eq. (12), the reduced density matrix of the  $N$ -qubit system at time  $t$  can be

expressed as

$$\begin{aligned}
 \rho_t &= \sum_{\mu_1, \mu_2, \dots, \mu_N} \left[ \otimes_{i=1}^N K_{\mu_i}(t) \right] \left[ \sum_j |\alpha_j|^2 |j\rangle\langle j| + \sum_{j \neq k} \alpha_j \alpha_k^* |j\rangle\langle k| \right] \left[ \otimes_{i=1}^N K_{\mu_i}^\dagger(t) \right] \\
 &= \sum_j |\alpha_j|^2 [\kappa_t^2 |j\rangle\langle j| + (1 - \kappa_t^2) |0\dots 0\rangle\langle 0\dots 0|] + \kappa_t^2 \sum_{j \neq k} \alpha_j \alpha_k^* |j\rangle\langle k| \\
 &= \kappa_t^2 \rho_0 + (1 - \kappa_t^2) |0\dots 0\rangle\langle 0\dots 0| \\
 &= P_t |\psi_0\rangle\langle\psi_0| + (1 - P_t) |0\dots 0\rangle\langle 0\dots 0|,
 \end{aligned} \tag{A5}$$

which exactly is the expression given by Eq. (16).

## Appendix B: The non-Markovianity measure

The non-Markovianity measure we employ here is based on the amount of information exchanged between the open system and its reservoir, which is defined as [57]

$$\Gamma = \max_{\rho_{1,2}(0)} \int_{\sigma(t)>0} dt \sigma(t). \tag{B1}$$

where  $\sigma(t)$  denotes the rate of change of the trace distance

$$\sigma(t) = \frac{d}{dt} \mathcal{D}[\rho_1(t), \rho_2(t)], \tag{B2}$$

and the trace distance is defined as

$$\mathcal{D}[\rho_1(t), \rho_2(t)] = \frac{1}{2} \text{Tr} |\rho_1(t) - \rho_2(t)|. \tag{B3}$$

Physically,  $\sigma(t) < 0$  ( $\Gamma = 0$ ) holds for all dynamical semi-groups and all time-dependent Markovian processes, while  $\sigma(t) > 0$  ( $\Gamma > 0$ ) for non-Markovian dynamics. It is should be noted that the maximum in Eq. (B1) is taken over all initial-state pairs of the system. Here we only consider the non-Markovianity in a pair of qubit-reservoir. Fortunately, it has been proved that, for the case of a qubit, the optimal initial-state pairs are always antipodal points on the Bloch sphere [58]. So we can assume  $\rho_{1,2}(0) = |\psi_{1,2}(0)\rangle\langle\psi_{1,2}(0)|$ ,  $|\psi_1(0)\rangle = \cos\theta|1\rangle + \sin\theta \exp(i\varphi)|0\rangle$  and  $|\psi_2(0)\rangle = \sin\theta|1\rangle - \cos\theta|0\rangle$

$\cos\theta \exp(i\varphi)|0\rangle$  with  $\theta \in [0, \pi/2]$ ,  $\varphi \in [0, 2\pi]$ . During the evolution time  $t$ , the optimal initial-state pair  $\rho_1(0)$  and  $\rho_2(0)$  evolve to the final-state pair  $\rho_1(t)$  and  $\rho_2(t)$ . According to Eq. (4), we can obtain the simple expression of the trace distance between two final states at time  $t$

$$\mathcal{D}(\rho_1(t), \rho_2(t)) = \sqrt{\cos^2(2\theta)P_t^2 + \sin^2(2\theta)P_t}. \tag{B4}$$

What is more, as the Bloch sphere governed by Eq. (4) is rotation symmetrical with respect to the  $z$  axis, the most strongest oscillating direction is probably either the pole direction or any direction in the equatorial plane. In other words, the optimal initial-state pair should be either  $\{|0\rangle, |1\rangle\}$  ( $\theta = 0$ ) or  $\{|+\rangle, |-\rangle\}$  ( $\theta = \pi/4$ ) with  $|\pm\rangle = (|0\rangle \pm e^{i\varphi} |1\rangle) / \sqrt{2}$  [59–61].

Substituting Eq. (B4) and Eq. (B2) into Eq. (B1) we can obtain the simple expression of the measure of non-Markovianity within the driving time  $\tau$

$$\Gamma = \max\{\Gamma_{\theta=0}, \Gamma_{\theta=\pi/4}\}, \tag{B5}$$

where the two non-Markovianity parameters  $\Gamma_{\theta=0}$  and  $\Gamma_{\theta=\pi/4}$  are defined by

$$\Gamma_{\theta=0} = \int_{0, \dot{P}_t > 0}^\tau \dot{P}_t dt, \tag{B6a}$$

$$\Gamma_{\theta=\pi/4} = \int_{0, \dot{P}_t > 0}^\tau \frac{\dot{P}_t}{2\sqrt{P_t}} dt. \tag{B6b}$$

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